Amplitude Equation for the Rosensweig Instability

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1. INTRODUCTION

Since its discovery in 1967 [1], the normal field or Rosensweig instability attracted the attention of experimentalists and theoreticians, alike. The phenomenon describes the transition of an initially flat ferrofluid surface to hexagonally ordered surface spikes as soon as an applied magnetic field exceeds a certain critical value. Ferrofluids are suspensions of magnetic nanoparticles in a suitable carrier liquid. One of the most prominent property is the superparamagnetic behavior in external magnetic fields, which accounts for the large magnetic susceptibility and the high saturation magnetization in rather low magnetic fields. If one starts to crosslink a mixture of a ferrofluid and a polymer solution with cross-linking agents, a superparamagnetic elastic medium, called ferrogel, is obtained [2]. As in usual ferrofluids, the initially flat surface of ferrogels becomes linearly unstable beyond a critical magnetic field [3]. A nonlinear analysis of the Rosensweig instability, however, turned out to be very complicated mainly due to the fact that the instability necessarily involves a deformable surface. In addition, the driving force is provided by the boundary conditions at the deformed surface and not by the bulk equations. A second problem arises from the fact that this instability is intrinsically dynamic in nature, although the linear threshold and the critical wavelength can be obtained in a static description (as an energetic minimum neglecting viscous effects). A nonlinear description, however, has to treat this instability as a breakdown of traveling surface waves.

2. RESULTS

We have succeeded in deriving from the basic hydrodynamic and magnetic equations and boundary conditions an amplitude and envelope equation for the Rosensweig instability. Starting from the linear solution, a systematic expansion of those nonlinear equations and boundary conditions in terms of the distance from the threshold is the standard procedure of a weakly nonlinear analysis. To adapt this method to the Rosensweig instability, the adjoint linear eigenvectors in the presence of a deformable surface are needed to satisfy Fredholm's theorem. This has been achieved recently [4] and is useful for other instability problems involving deformable surfaces, like the Marangoni instability. This enabled us to perform the usual expansion procedure for the full dynamic problem up to third order, which in the limit of vanishing frequency gives the desired amplitude and envelope equation for the stationary instability. It contains cubic and quadratic nonlinearities as well as first and second order time derivatives of the pattern amplitudes and will be discussed in the talk. A comparison will be made to corresponding results obtained by the so-called energy method [6], which is approximative and not systematic.

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