I discuss our recent exciting and fascinating results on systems where quadrupolar order is embedded in an octupolar field.

Helmut Brand, Harald Pleiner and myself came up with banana liquid crystals [1]. We found that even though these objects were achiral (no asymmetric carbons) they could, by symmetry, have a polarization, $P$, when stacked in layers. To our surprise, when achiral banana liquid crystals were first synthesized [2], they spontaneously exhibited ambidextrous chirality – both left and right-handed structures were observed. Ambidextrous chirality is unknown in quadrupolar ($Q_{ij}$) liquid crystals formed by rod or disc liquid crystals (Fig. 2) [3].

Before achiral banana-shaped liquid crystals had been synthesized, Leni Fel went beyond quadrupolar liquid crystals ($Q_{ij}$) to consider liquid crystal phases with tetrahedratic (octupolar) order, $T_{ijk}$ [4] (Fig. 3). Tetrahedratics lack inversion symmetry, have no hand and are optically isotropic. After banana LCs were synthesized [4,5], Radzihovsky and Lubensky argued [6], that banana symmetry required a vector order parameter, $P$ [1], a quadrupolar order parameter, $Q_{ij}$ [3], and also, Fel’s octupolar order parameter, $T_{ijk}$ [4]

But symmetry allows free energy gradient term in $Q_{ij}$ and $T_{ijk}$ to couple at lower order than spatially homogenous ones, rather like cholesterics [7]. This lowest order coupling term, $T_{ijk}Q_{ijk}$, accounts for ambidextrous chirality in achiral banana liquid crystals by reducing the energy of ambidextrous twist in both fields below that of the uniform state. It also explains the spontaneous appearance of splay-bend defects in a biaxial Bouligand arceau (Fig. 4) with $Q_{ij}$ embedded in $T_{ijk}$: a 5-stranded braid – 4 strands for $T_{ijk}$ and 1 for $Q_{ijk}$.

**References**


